B.Sc Part I

To find the necessary and sufficient condition that the two circles may cut one another orthogonally

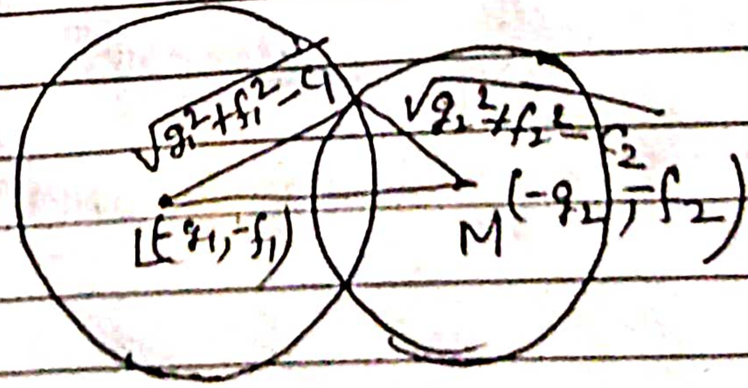
Let the two circles be

$x^2+y^2+2g_1x+2f_1y+c_1=0$ and

$x^2+y^2+2g_2x+2f_2y+c_2=0$ which intersect each other orthogonally at P

Here the centres L and M of the circles are respectively $(-g_1,-f_1)$ and $(-g_2,-f_2)$, and the radii LP and PM are $\sqrt{g_1^2+f_1^2-c_1}$ and $\sqrt{g_2^2+f_2^2-c_2}$

Now



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Now $LM^2 = (g_1 - g_2)^2 + (f_1 - f_2)^2$

$= g_1^2 + g_2^2 - 2g_1g_2 + f_1^2 + f_2^2 - 2f_1f_2$

Since the circles cut each other orthogonally at P.

therefore $\angle LPM = \pi/2$

$$LP^2 + PM^2 = LM^2$$

Or,
$g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2$

$= g_1^2 + g_2^2 - 2g_1g_2 + f_1^2 + f_2^2 - 2f_1f_2$

Or, $2g_1g_2 + 2f_1f_2 = c_1 + c_2$